Alcuni limiti notevoli

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^x = e \qquad \text{con } 1+\frac{1}{x}>0 \Leftrightarrow \frac{x+1}{x}>0 \Leftrightarrow x\in\left]-\infty,-1\right[\cup\left]0,+\infty\right[$$

$$\lim_{x\to+0}(1+x)^{\frac{1}{x}} = e \qquad \text{con } x+1>0 \Leftrightarrow x\in\left]-1,+\infty\right[-\{0\}$$

$$\lim_{x\to+0}\frac{\log(1+x)}{x}=1 \qquad \text{con } x+1>0 \Leftrightarrow x\in\left]-1,+\infty\right[-\{0\}$$

$$\lim_{x\to+0}\frac{\log_a(1+x)}{x}=\log_a e \qquad \text{con } x+1>0 \Leftrightarrow x\in\left]-1,+\infty\right[-\{0\}$$

$$\lim_{x\to+0}\frac{a^x-1}{x}=\log_e a \qquad \text{con } x\in R-\{0\}$$

$$\lim_{x\to+0}\frac{e^x-1}{x}=\log_e e \qquad \text{con } x\in R-\{0\}$$

$$\lim_{x\to+0}\frac{senx}{x}=1 \qquad \text{con } x\in R-\{0\}$$

$$\lim_{x\to+0}\frac{tgx}{x}=1 \qquad \text{con } x\in R-\left(\bigcup_{h\in Z}\left\{\frac{\pi}{2}+h\pi\right\}\cup\{0\}\right)$$

$$\lim_{x\to+0}\frac{1-\cos x}{x}=0 \qquad \text{con } x\in R-\{0\}$$

$$\lim_{x\to+0}\frac{1-\cos x}{x^2}=\frac{1}{2} \qquad \text{con } x\in R-\{0\}$$

$$\lim_{x\to+0}\frac{ar\cos enx}{x}=1 \qquad \text{con } x\in[-1,1]-\{0\}$$

$$\lim_{x\to+0}\frac{arctgx}{x}=1 \qquad \text{con } x\in R-\{0\}$$

Sia $f : X \to R$, sia $\lim_{x \to +x_0} f(x) = 0$, sia $f(x) \neq 0 \quad \forall x \in X - \{x_0\}$

Allora si ha che tutti i limiti di cui sopra, nella funzione composta $g(f(x))$, hanno stessa convergenza:

$$\lim_{x \to +x_0} (1 + f(x))^{\frac{1}{f(x)}} = e$$

$$\lim_{x \to +x_0} \frac{\log(1 + f(x))}{f(x)} = 1$$

$$\lim_{x \to +x_0} \frac{\log_a(1 + f(x))}{f(x)} = \log_a e$$

$$\lim_{x \to +x_0} \frac{a^{f(x)} - 1}{f(x)} = \log_e a$$

$$\lim_{x \to +x_0} \frac{e^{f(x)} - 1}{f(x)} = \log_e e$$

$$\lim_{x \to +x_0} \frac{senf(x)}{f(x)} = 1$$

$$\lim_{x \to +x_0} \frac{tgf(x)}{f(x)} = 1$$

$$\lim_{x \to +x_0} \frac{1 - \cos f(x)}{f(x)} = 0$$

$$\lim_{x \to +x_0} \frac{1 - \cos f(x)}{(f(x))^2} = \frac{1}{2}$$

$$\lim_{x \to +x_0} \frac{ar\cos enf(x)}{f(x)} = 1$$

$$\lim_{x \to +x_0} \frac{arctgf(x)}{f(x)} = 1$$