COURSE OF STUDY Physisc L 30

ACADEMIC YEAR *2023-2024*

ACADEMIC SUBJECT Calculus I

General information	
Year of the course	First year
Academic calendar (starting and ending date)	22/09/2023- 22/12/ 2023
Credits (CFU/ETCS):	8
SSD	Calculus
Language	Italian
Mode of attendance	no

Professor/ Lecturer	
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Department and address	*Aula A, Dipartimento Interateneo di Fisica*
Virtual room	*Microsoft Teams (lessons in presence with online notes))*
Office Hours (and modalities: e.g., by appointment, on line, etc.)	*Single, on demand on Microsoft Teams or more student in classroom after the lessons*

Work schedule			
Hours			
Total	Lectures	Hands-on (laboratory, workshops, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
200	40	45	115
CFU/ETCS			
8	5	3	

Learning Objectives	
Course prerequisites	It is the first exam on a mathematical topic of the first year, no preliminary knowledge is required other than that required for access to the degree course, however concerning the pre-course called Introduction to Mathematical Analysis whose notes are in the course channel. These prerequisites include: Analytical Geometry, Logical and set theory language, Operations between polynomials

Teaching strategie	Lectures with slides that are carried out in the classroom so that explanation and understanding align. The slides created in the classroom are distributed at the end of the lesson on the Microsoft Teams platform. Classroom exercises with Prof. Alessandro Palmieri with handouts and proposed homeworks.
Expected learning outcomes in terms of	
Knowledge and understanding on:	At the end of the course the student will know ○ Natural numbers ○ The real line ○ Concept of function, limits, continuity, derivatives, ○ Successions, series ○ Derivation and integration tools

	○ Proofs of the most important theorems of these topics
Applying knowledge and understanding on:	○ Review of basic knowledge. ○ Make the discrete mathematical world interact with the continuous one ○ Independently prove other theorems of the real line ○ Compare course topics with some of the topics in first-year physics courses
Soft skills	• *Making informed judgments and choices* ○ Comparison between various demonstrations. ○ Treatment of incoming data and critical analysis of the results in solving numerical problems • *Communicating knowledge and understanding* ○ knowing how to define, state and prove mathematical results ○ knowing how to explain to others your own resolution of an exercise • *Capacities to continue learning* ○ Acquire a study method that allows you to consult mathematics texts and keep the results in mind ○ Knowing how to choose exercises from the texts ○ Study theory and exercises at the same time
Syllabus	
Content knowledge	*1. Real numbers* Hints of logic. Set theory: belonging, inclusion, union, intersection, complementary set, Cartesian product. What is an order relation. Natural numbers N, integers Z, rational numbers Q and their structures. No rational number has square 2. Finite, infinite and countable sets. Principle of induction. Bernoulli inequality. The real line, the intervals. Numerical sets: sup and inf, maximum and minimum. Field axioms of real numbers. The absolute value of a real number. Equivalent forms of the completeness axiom. Density of Q and its complementary in R. Archimedean property. *2. Elementary functions:* What is a function. Injective, surjective, bijective functions. Composition of functions, invertible functions and their inverse. Restriction and extension of a function. Direct and inverse image. The graph of a real function. Limited functions. Monotony, symmetries and periodicity of a function. Construction of some elementary functions, properties and graphs. Elementary operations on function graphs. Rational, irrational and transcendent inequalities. Theorem: Any strictly monotone real function is ingective. *3. Numerical sequences:* What is a sequence. Regular sequences. Uniqueness of the limit. The expanded set R. Indeterminate forms. Operations on the limits of succession. Theorem: every convergent sequence is bounded. Theorems: sign permanence and conservation of inequalities for sequences. Comparison and ratio theorems for limit of sequences. Fundamental theorem on the limit of monotone sequences. Newton's binomial. Number of Napier. Remarkable limits of succession, scale of infinities. Subsequences and related theorem. Bolzano-Weierstrass theorem. Sequences defined by recurrence. *4. Function limits:* Accumulation points and closed sets. Geometric examples (exhaustion) Function limits defined by sequences. Limit from right and left. Algebraic and comparison results for limits of functions, which are deduced from the same results for limits of sequences. Epsilon-delta rewriting of function limits. Operations on limits. Comparison theorems. Limits of elementary functions. Theorem: every convergent function is locally bounded. Notable limits. Infinites and infinitesimals and their properties. Principle of elimination of negligible terms. Horizontal and oblique asymptotes of a function.

	5. Continuous functions: Continuous functions and their elementary properties. The permanence of the sign theorem. When a function is not continuous at one point? Jump points, oscillations, continuity extension, vertical asymptotes. Continuous functions on intervals. The elementary functions are continuous in their domain. Zeros theorem. Weierstrass theorem. Intermediate value theorems. Existence of inverse of elementary functions. Link between monotony, continuity and invertibility. Continuity of the inverse function on intervals. Uniform continuity. Lipschtzian functions. Cantor's theorem. *6. Differential calculus:* Derivative of a function of a real variable. Continuity of differentiable functions. Theorem on the derivative of operations between functions (sum, product, quotient, composition) Derivative of the inverse function. Derivability of elementary functions. Angular points, cusp points. Local maximum and minimum points, critical points. Fermat's theorem. Rolle, Cauchy, Lagrange theorems. Monotony criteria. Functions with null derivative. Function with bounded derivative. De l'Hospital's theorem. Convexity for differentiable functions. Convex functions on an interval. Link between second derivative and convexity. Regularity of convex functions. Inflection points. Sufficient conditions for the existence of relative maximums, minimums. Taylor's formula with the remainder of Peano. Taylor's formula with Lagrange's remainder. Taylor expansions for elementary functions. Applications of Taylor's formula to classify maxima, minima and inflections. Study of the graph of a function. Examples of geometric nature (tangent line) and kinematics (speed, acceleration). *7. Integral calculation:* Partition of an interval. Upper and lower integral sums. Integrability according to Riemann. Characterization of integrable functions. Elementary properties of the definite integral. Integrability theorem of continuous and monotone functions. Mean theorem. Integral functions. Primitive and indefinite integral. Fundamental theorem of integral calculus. Structure theorem of the set of primitives of a continuous function. Torricelli's theorem. Methods for calculating indefinite integrals for rational functions. Integration by parts. Integration by substitution. *8. Numerical series:* Definition of series and sum of series. The telescopic series (Mengoli series). The geometric series. Application of series to the decimal representation of real numbers. The harmonic series. Necessary condition for the convergence of a series. The character of a series does not change by altering a finite number of terms. Series with non-negative terms, dichotomy theorem. Simple comparison criteria. Criterion of the asymptotic comparison. The generalized harmonic series. Criterion of infinitesimals. Root criterion, relationship criterion. Absolutely convergent series are convergent. Alternating series. Leibnitz criterion for alternating series. The harmonic series with alternating sign. Interlocking sum. *9. Generalized integrals* Generalized integrals: integration of a function on a ray or of an unlimited function on a limited interval. The integral criterion for numerical series. Application to the generalized harmonic series. The Gamma function of Euler and sinc function.
Texts and readings	Theory: M. Bertsch, A. Dall'Aglio, L. Giacomelli, Epsilon 1, MacGraw Hill Exercise: P. Marcellini & C. Sbordone -Elementi di Analisi Matematica 1– Liguori Editore, Napoli.
Notes, additional materials	The program is related only to some sections of the texts indicated. The previous texts are only recommended, it is good that each student consults other texts of Mathematical Analysis at the Uniba libraries looking for the most appropriate one for their starting level.

	In particolare si segnalano i seguenti: Theory: E. Acerbi, G. Buttazzo – Primo corso di Analisi Matematica – Pitagora M. Bramanti, C.D. Pagani, S. Salsa – Analisi Matematica 1 - Zanichelli Exercise M. Bramanti -Esercitazioni di Analisi Matematica- Esculapio A. Alvino, C. Carbone, G. Trombetti -Esercitazioni di matematica Vol 1/1 Vol 1/2 - Liguori Editore.
Repository	Teacher's notes: https://www.sandralucente.it/didattica/appunti-lezioni Lessons notes on Microsoft Teams

Assessment	
Assessment methods	Two ongoing tests passed or a final written test. The written tests last at least two hours. Then oral exam lasting at least thirty minutes. During the written test only the non-graphical scientific calculator is allowed. The student who any device connected to the internet is removed from the written test. The student must book on esse3 both the written test and the oral exam. The results are provided on the same platform. The oral exam can be held for the entire session in which the written test is passed or for the entire academic year if the ongoing tests are passed.
Assessment criteria	• *Knowledge and understanding* Knowing how to consult your lesson notes and compare them with texts, discuss doubts and ideas deriving from them with the teacher and possibly with classmates. • *Applying knowledge and understanding* Knowing how to plot graphs of elementary functions, be familiar with equations and inequalities • *Autonomy of judgment* Knowing how to evaluate the coherence of a logical reasoning. Knowing how to choose the appropriate mathematical tools to solve a given problem • *Communicating knowledge and understanding* During the written test it is verified that the student knows the techniques for the study of function, the resolution of integrals, the discussion on the existence of limits and on the convergence of series. During the oral exam it is verified that the student knows theorems, definitions, examples (therefore exercises) and counterexamples and knows how to correlate them. • *Communication skills* Ability to write an exercise paper that topics the steps carried out; *ability to communicate their knowledge in correct mathematical language during the oral exam* • *Capacities to continue learning* Capacity in consulting textbooks, in finding logical links and solving exercises.
Final exam and grading criteria	*The final grade is assigned in thirtieths. The exam is passed when the mark is greater than or equal to 18.* The written test is passed if the student is familiar with each of the four exercises proposed. The oral exam is passed if the student proves a theorem at the request of the teacher, knows how to expose the definitions and provide reasons for the hypotheses of the theorems. If the student has completely

	omitted the study of a part of the program, regardless of the learning of the remaining part, the exam is not passed. The final grade depends on the mistakes made in the written test and on the ability to expose the oral. Praise is given to students who, in addition to the deep knowledge of the program, are able to support a critical discussion on examples and counterexamples to the various theorems.
Further information	