General information		
Academic subject	GEOMETRY	
Degree course	Physics	
Academic Year	1st	
European Credit Transfer and Accumulation System (ECTS)		9
Language	Italian	
Academic calendar (starting and ending date)	1st semester (19th September 2022 – 16th December 2022)	
Attendance	According to didactic regulations	

Professor/ Lecturer	
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Department and address	Department of Mathematics, 4th floor, room 5
Virtual headquarters	Teams
Tutoring (time and day)	Tutoring takes place by appointment to be agreed by email, in person or remotely via Microsoft Teams.

Syllabus	
Learning Objectives	Acquiring fundamental notions in linear algebra, affine and Euclidean geometry: matrix calculus and systems of linear equations, vector spaces and linear maps, eigenvalues, eigenvectors and diagonalization of endomorphisms, scalar products, affine spaces, Euclidean spaces.
Course prerequisites	Basic mathematical knowledge: polynomials, first and second degree equations and inequalities, fundamental theorems of Euclidean geometry, elements of trigonometry, elements of analytic geometry in dimension 2.
Contents	**Basic set theory.** Union and intersection of sets, complement of a set, the powerset of a set. Ordered pairs and n-tuples. Cartesian product. Relations. Order relations. Equivalence relations, equivalence classes and quotient set. Functions. Image and preimage of sets. Surjective, injective and bijective functions. Function composition. Inverse function. **Algebraic structures.** Binary operations. Associativity, commutativity, identity element and inverses. Groups and subgroups. Rings. Fields and subfields. Complex numbers and field structure. Conjugate and modulus of a complex number. Polynomial ring in x over a field K. Properties of polynomials. **Vector spaces.** Vector spaces, properties and examples. The space of geometric vectors. Linear subspaces. Intersection, sum and direct sum of subspaces. Supplementary subspaces. Linear combinations of vectors. Vector space spanned by n vectors. Finitely generated vector spaces and systems of generators. Linearly dependent and linearly independent vectors. Bases. Components of a vector with respect to a basis. Existence of bases: procedure to find bases. Dimension of a finitely generated vector space. Dimension of linear subspaces. Extension of linearly independent vectors to a basis. Grassmann identity.

Matrices and systems of linear equations. The vector space of matrices with m rows and n columns over a field K. The transpose of a matrix. Square matrices, symmetric, skew-symmetric, diagonal matrices. Trace of a square matrix. Matrix product. Determinant of a square matrix: definition and properties. Invertible matrices and inverse matrix. The group GL(n, K) and its subgroups. Orthogonal matrices. Rank of a matrix: definition and properties. Matrix associated to a set of vectors with respect to a basis. Change of basis matrix. Systems of m linear equations in n variables. Cramer's systems. Rouché-Capelli theorem. Homogeneous systems. General method for finding the solution set of a linear system.

Linear maps. Linear maps between vector spaces. Characterization and properties. Kernel and image of a linear map. Characterizations of surjective or injective linear maps. Existence and uniqueness of linear maps. Isomorphisms. Matrices associated with a linear map. Eigenvectors, eigenvalues and eigenspaces of an endomorphism. Diagonalizable endomorphisms: definition and characterization. Similar matrices. Diagonalizable matrices. Characteristic polynomial. Algebraic and geometric multiplicity of an eigenvalue. Criterion of diagonalizability of endomorphisms.

Euclidean vector spaces. Orientation of a real vector space. Euclidean vector spaces. Scalar product. Standard scalar product on R^n. The norm of a vector. Convex angle between two non-zero vectors. Parallel vectors. Orthogonal vectors. Systems of orthogonal vectors. Orthonormal bases. Change of orthonormal basis matrix. Gram-Schmidt theorem. Orthogonal complement of a linear subspace. Unitary operators: definition, properties and examples. Unitary operators and orthogonal matrices.

Affine Spaces. Affine space associated to a vector space. Affine frames and coordinate systems. Affine subspaces and their direction. Affine subspace generated by k points. Affinely independent points. Parametric and cartesian equations of an affine subspace. Parallel subspaces. Intersection of affine subspaces. Change of affine frames.

Affine geometry in dimension 2. Coordinate axes. Parametric equations and cartesian equation of a line. Parallel lines and intersection of lines.

Affine geometry in dimension 3. Coordinate axes and planes. Parametric and cartesian equations of a plane. Parametric and cartesian equations of a line. Parallel lines. Parallelism between a line and a plane. Parallelism between planes. Coplanar lines.

Euclidean spaces. Euclidean space associated to a Euclidean vector space. Cartesian frames and cartesian coordinates. Change of cartesian frames. Distance between two points. Convex angles between two lines. Orthogonal lines.

Euclidean geometry in dimension 2. Orthogonal lines.

Euclidean geometry in dimension 3. Orthogonal lines, orthogonality between a line and a plane. Convex angles between two planes and orthogonality. Equation of a sphere.

Isometries of a Euclidean space of dimension n. Definition, characterization and geometric properties of isometries. Equations of an isometry with respect to a cartesian frame. Examples: translations and rotations.

	Euclidean conics. Equation of a conic. Canonical form of a Euclidean conic. Hints on Euclidean quadrics.
Books and bibliography	– E. Sernesi, Geometria 1, Bollati Boringhieri. – E. Abbena, A.M. Fino, G.M. Gianella, Algebra lineare e geometria analitica, Aracne. – Facchini, Algebra e Matematica Discreta, Zanichelli. – Lecture notes and exercise sheets available on Microsoft Teams.
Additional materials	

Work schedule			
Total	Lectures	Hands on (Exercises)	Out-of-class study hours/ Self-study hours
Hours			
225	56	30	139
ECTS			
9	7	2	

Teaching strategy	Lectures and exercises. Exercise sheets will be provided.
Expected learning outcomes	
Knowledge and understanding on:	Acquiring fundamental concepts in linear algebra, dealing with vector spaces, linear maps and scalar products, and the basics of affine and Euclidean geometry. Acquiring basic mathematical proof techniques.
Applying knowledge and understanding on:	The acquired theoretical knowledge is applied in solving problems in linear algebra and geometry, where students are particularly concerned with matrix calculus, systems of linear equations, bases and dimension of vector spaces, kernel and image of linear maps, diagonalization of endomorphisms or matrices, description of subspaces (linear, affine or Euclidean) through parametric or Cartesian equations.
Soft skills	• *Making informed judgments and choices* Ability to analyse the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge. • *Communicating knowledge and understanding* Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyse and solve problems. • *Capacities to continue learning* Acquiring suitable learning methods, supported by text consultation and by solving the exercises and questions periodically suggested during the course.

Assessment and feedback	

Methods of assessment	Oral exam, including exposition of definitions, statements and proofs, and solving exercises.
Evaluation criteria	*At the end of the course the following points will be evaluated:* ● *Knowledge and understanding*: Knowledge of the fundamental concepts in linear algebra, affine and Euclidean geometry, together with the capacity to state and prove related properties; capacity to show the acquired notions in specific examples; ● *Applying knowledge and understanding*: Knowledge of how to use the acquired theoretical notions in solving exercises of linear algebra and geometry, including: matrix calculus, systems of linear equations, bases and dimension of vector spaces, kernel and image of linear maps, diagonalization of endomorphisms or matrices, parametric and cartesian equations of subspaces (linear, affine or Euclidean), with the description of related properties. ● *Autonomy of judgement*: Capacity in evaluating the consistency of the logical arguments used in a proof. Problem solving skills, coherently with the acquired theoretical knowledge. ● *Communication skills*: Capacity in the exposition of definitions, statements and proofs, and in presenting solutions of exercises in suitable mathematical language and formalism. ● *Capacities to continue learning*: Capacity in consulting textbooks, in finding logical links and solving exercises.
Criteria for assessment and attribution of the final mark	The final grade is out of thirty. The exam is passed when the grade is greater than or equal to 18/30.
Additional information	