UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

Dipartimento di Economia e Finanza

Course of Study: Statistics

Academic Year: 2024/2025

Academic Subject: Linear algebra

General information	
Year of the course	Second Year
Academic calendar (starting and ending date)	First semester: from Sep. 9th 2024 to Dec. 13th 2024
Credits (CFU/ETCS):	6 CFU
SSD	MAT/03
Language	Italian
Mode of attendance	Attendence is not mandatory

Professor/ Lecturer	
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Telephone	
Department and address	Economics and Finance, Largo Abbazia Santa Scolastica - 70124 Bari
Virtual room	Microsoft Teams code x79mw1p
Office hours	By appointment, to be requested via email

Work schedule			
Hours			
Total	Lectures	Hands-on (laboratory, workshops, working groups, seminars, field trips)	Out-of-class study hours /Self-study hours
150	42	0	108
CFU/ETCS			
6			

Learning Objectives	The course aims to provide the basic tools of matrix calculus and linear transformations. These tools will enable the student to successfully attend the other classes and the subsequent professional statistician activity. The lessons are aimed at enhancing and refining the logical deductive skills and the critical sense of the student, to get him used to expressing himself with precision and language properties.
Course prerequisites	The basic notions of a first year calculus course.

Teaching strategie	Theoretical lectures and exercises.
Expected learning outcomes in terms of **Knowledge and understanding on:** **Applying knowledge and understanding on** **Soft skills**	The main notions and the most important results of matrix calculus and linear transformations. The student will have to develop the ability to solve problems by applying the theorems, tools and methods learned in class. The student must be able to critically evaluate different operational solutions in order to identify the most suitable for the objectives to be pursued.
Syllabus	
Content knowledge	**Vectors in $\mathbb{R}^n$.** Cartesian coordinate system. Plane vectors and vectors in a three-

dimensional space. Operations between vectors. Vectors in $\mathbb{R}^n$; sum of two vectors, product of a vector and a scalar, scalar product of two vectors and related properties. Orthogonal vectors in $\mathbb{R}^n$. Euclidean norm and distance in $\mathbb{R}^n$.

Matrices. Matrices of type *m* x *n*. Square matrices. Upper or lower triangular matrices. Diagonal matrices. Block matrices. Transposed matrix. Operations between matrices: sum of two matrices, product of a matrix by a scalar, matrix product. Elementary operations on the rows of a matrix. Pivot step. Gauss Jordan algorithm.

Determinants. Determinant of a square matrix and related properties. Calculation of the determinant with the Gauss Jordan algorithm and with Laplace's rule. Invertible square matrices and relative inverse. Calculation of the inverse matrix of an invertible square matrix with the Gauss-Jordan algorithm. Cramer's theorem. Calculation of the inverse matrix of an invertible square matrix with the addition matrix method.

Vector spaces. Definition of vector spaces. Subspaces. Subspace spanned by a finite set of vectors. Linearly dependent and linearly independent vectors system. Generators and basis of a subspace. Examples and properties. Dimension. Rank of a matrix. Computing the rank of a matrix by using the Gauss-Jordan algorithm. Straight lines, planes and hyperplanes. Affine subspaces. Parametric and Cartesian formula of an affine straight line and of an affine plane in $\mathbb{R}^n$. Parametric and Cartesian formula of the straight line passing through two points and parametric formula of the plane passing through three points.

Linear systems. Rouché Capelli theorem. Solving a linear system (homogeneous and nonhomogeneous) of *m* equations in *n* unknowns using the Gauss-Jordan algorithm. The set of the solutions of a homogeneous linear system $Ax=0$ is a subspace of dimension $n-\text{car}(A)$. The set of solutions of an inhomogeneous linear system $Ax=b$ is an affine subspace of dimension $n-\text{car}(A)$ parallel to the subspace of solutions of the homogeneous system $Ax=0$.

Linear transformations. Linear transformations between vector spaces: examples and counterexamples. Fundamental properties of linear transformations. Kernel and image of a linear transformation. Dimension theorem. Linear transformation associated with a matrix. Change of coordinates. Matrix of a linear transformation with respect to given bases. Matrix of a linear transformation of a vector space in itself with respect to a basis. Similar matrices.

Eigenvalues and eigenvectors. Eigenvalues and eigenvectors of a square matrix. Algebraic multiplicity and geometric multiplicity. Similar matrices, diagonalizable matrices. Meaning of diagonalizability. Necessary and sufficient conditions for diagonalizability. Properties of the eigenvalues and eigenvectors of a symmetric square matrix. A symmetric square matrix can be diagonalized by an orthogonal matrix.

Quadratic forms. Quadratic form associated to a symmetric square matrix. Signature of a quadratic form. Congruent symmetric matrices. Sylvester's law of inertia. Reduction in canonical form of a symmetric square matrix. Three methods for computing the signature of a quadratic form: 1) using the sign of the eigenvalues, 2) using the sign of the coefficients of the characteristic

	polynomial, 3) using the sign of the principal minors. **Applications.** Covariance matrices and their inverses. Pearson's inequality. Linear differential equations.
Texts and readings	M. Brabanti, C.D. Pagani, S. Salsa: Matematica, Calcolo infinitesimale e algebra lineare, Zanichelli.
Notes, additional materials	
Repository	

Assessment	
Assessment methods	Written and successive oral exam. The written exam consists in various exercises on different topics of the course. The oral exam consists in the discussion of the written test and the verification of knowledge on additional topics that are not covered by the written test: the definitions of the concepts and the statements of the theorems covered in the course are required. The proofs of the main results are also required. In order to be admitted to the oral exam it is necessary to pass the written exam.
Assessment criteria	The following aspects are equally valued • the knowledge *and understanding* of the notions of the course, • the ability to apply the notions learned to the resolution of exercises, • the a*utonomy of judgment,* • the ability to expose with precision and rigour, • the c*apacities to continue learning.*
Final exam and grading criteria	The final mark is a global evaluation of the written and oral exams.
Further information	