UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

DIPARTIMENTO INTERUNIVERSITARIO DI FISICA

General information		
Academic subject	Mathematical Methods of Physics	
Degree course	Physics	
Academic Year	1st	
European Credit Transfer and Accumulation System (ECTS)		6
Language	English	
Academic calendar (starting and ending date)	1st semester: Last week of September – Third week of December	
Attendance	Free	

Professor/ Lecturer	
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Department and address	*Dipartimento Interateneo di Fisica, office 182*
Virtual headquarters (Microsoft Teams code)	
Tutoring (time and day)	Students are invited to send an e-mail to arrange individual or group meetings

Syllabus	
Learning Objectives	Acquisition of advanced mathematical methods for modern physics
Course prerequisites	Real and complex analysis, Fourier transform, Distribution theory, Quantum mechanics
Contents	***Metric spaces.*** *Definition. Examples. Open sets, closed sets, neighborhoods. Topological spaces. Continuous mappings. Dense sets, separable spaces. Convergent and Cauchy sequences. Completeness. Examples. Completion of a metric space.* ***Banach spaces.*** *Vector spaces. Normed spaces. Completeness and Banach spaces. Examples: finite dimensional spaces, sequence spaces, function spaces. Bounded linear operators. Continuity and boundedness. BLT theorem. Continuous linear functionals and dual spaces. Banach space of bounded linear operators. Examples.* ***Introduction to measure theory.*** *Lebesgue integral. Sigma algebras and Borel measures. Measurable functions. Dominated and monotone convergence. Fubini theorem. Examples: absolutely continuous measure, Dirac measure, Cantor measure. Lebesgue decomposition theorem.* ***Hilbert spaces.*** *Inner product. Euclidean and Hilbert spaces. Orthogonality, Pythagorean theorem. Bessel and Cauchy-Schwarz inequalities. Triangular inequality. Parallelogram law and polarization identity. Examples. Direct sum. Projection theorem. Riesz-Fréchet lemma. Orthonormal systems and Fourier coefficients. Orthonormal bases and Parseval's relation. Gram-Schmidt orthogonalization procedure. Isomorphism with l^2. Tensor product and product bases.* ***Linear operators on Hilbert spaces.*** *$C*$-algebra of bounded operators. Normal, self-adjoint, unitary and projection operators. Baire's category theorem. Uniform boundedness principle. Uniform, strong and weak convergence. Some quantum mechanics. Unbounded operators. Adjoint. Symmetric and self-adjoint operators. Examples: multiplication and derivation operators. Essentially self-adjoint operators. Fundamental criteria of self-adjointness and essentially self-adjointness. Graph, closure and inverse of an operator. Self-adjoint extensions of positive operators. Example: kinetic energy in a segment. Self-adjointness of observables.* ***Spectrum and dynamics.*** *Resolvent operator, resolvent set and spectrum. Examples: position and momentum operators. First resolvent formula and analytic properties. Neumann series. Spectrum and Weyl sequences. Spectrum and eigenvalues of the inverse. Spectrum of self-adjoint, unitary and projection operators. Projection-valued measures and resolution of the identity. Integration on PVM of bounded functions. Expectation value of the resolvent. Spectral family of a self-adjoint operator and spectral theorem. Functional calculus. Spectral projections and spectral types. Quantum dynamics and unitary evolution groups.*

	Energy conservation. Stone's theorem. Return and transition probability. Riemann-Lebesgue and Wiener Lemmas. Spectral types and return probability. Pure point spectrum and quasi periodic orbits. RAGE theorem.
Books and bibliography	- M. Reed, B. Simon, Methods of Modern Mathematical Physics, Vol. 1, Academic Press, New York, 1980 - G. Teschl, Mathematical Methods in Quantum Mechanics, American Mathematical Society, Providence, 2009 - Lecture notes
Additional materials	Available online at http://www.ba.infn.it/~facchi/Sito/Lectures.html

Work schedule			
Total	Lectures	Hands on (Laboratory, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
Hours			
150	24	45	81
ECTS			
6	3	3	

Teaching strategy	
	Lectures and exercise sessions

Expected learning outcomes	
Knowledge and understanding on:	The student will acquire knowledge of the advanced mathematical techniques commonly used in fundamental and applied research in physics. In particular, a knowledge of the mathematical structures of functional analysis and the theory of operators on Hilbert spaces, necessary for understanding advanced problems of modern physics.
Applying knowledge and understanding on:	The student will acquire knowledge of general and advanced analytical and approximation techniques for understanding quantum phenomena and solving problems in quantum mechanics and quantum field theory.
Soft skills	• **Making informed judgments and choices** Within the mathematical methods of physics, the student will be able to identify the best mathematical strategy for tackling specific physical problems. • **Communicating knowledge and understanding** The student will acquire mastery of the mathematical lexicon of modern physics and of quantum physics. • **Capacities to continue learning** The student will develop an attitude to the continuous updating of mathematical techniques and skills in physics research.

Assessment and feedback	
Methods of assessment	Oral exam; written exercise
Evaluation criteria	Capability to use techniques and solve problems introduced in the course. Adequate comprehension and global knowledge of concepts and arguments described throughout the course.
Criteria for assessment and attribution of the final mark	Written exercise (50%). Oral exam (50%)
Additional information	